

3.5

Collect Like Terms



Recreational vehicles can be a lot of fun. However, they sometimes require costly repairs. Suppose you are faced with a repair bill involving parts and labour charges. How can you use algebra to simplify the information?

Example 1 Build and Simplify an Algebra Tile Model

Crystal bought a used personal watercraft with her summer earnings. Unfortunately, it needs some repairs before she can use it.

- Model each repair using algebra tiles and write an expression to describe it.
- Find a simplified expression for both repairs.
- Calculate the total repair cost if the mechanic charges \$50/h for labour.

Item	Cost for Parts(\$)	Labour (h)
Alternator	200	4
Fuel Pump	100	3

Solution

- Let each unit tile represent \$100 and each x-tile represent an hour's labour charge.

Alternator



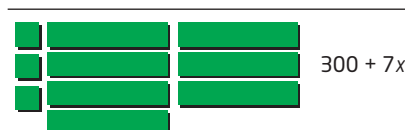
$200 + 4x$ expresses the cost to repair the alternator.

Fuel Pump



$100 + 3x$ expresses the cost to repair the fuel pump.

- Find a **simplified** expression for both repairs by adding these expressions.



Making Connections

You will study relations such as this in **5.2 Partial Variation**.

Literacy Connections

To **simplify** an expression is to find an equivalent, simpler, and shorter expression.

Original expression:
 $200 + 4x + 100 + 3x$

Simplified expression:
 $300 + 7x$

- c) The mechanic charges \$50/h. Substitute 50 into the expression and evaluate it.

$$\begin{aligned} & 300 + 7x \\ &= 300 + 7(50) \\ &= 300 + 350 \\ &= 650 \end{aligned}$$

The total repair cost for Crystal's personal watercraft is \$650.

In Example 1 you added fixed costs—the cost of the parts:

$$200 + 100 = 300$$

You also added variable costs—the cost of the labour:

$$4x + 3x = 7x$$

When you add all the fixed costs, or add all the variable costs, you are adding **like terms**.

like terms

- terms that have identical variables

Example 2 Identify Like and Unlike Terms

Identify the like terms in each group.

- a) $2x$, 2 , $5x$, $3xy$, $3x$
 b) $4a^2$, $2a$, $-a^2$, a^3 , a^2b
 c) $3u^2$, $-4uv$, 6 , $2u^2$, $\frac{1}{2}v$, -5

Solution

- a) Like terms have identical variable parts.

$$2x, 2, 5x, 3xy, 3x$$

$2x$, $5x$, and $3x$ all have x as the variable. They are like terms.

$2x$ and 2 are unlike terms because the second term has no variable.

$2x$ and $3xy$ are unlike terms because the second term includes a y .

- b) $4a^2$, $2a$, $-a^2$, a^3 , a^2b

$4a^2$ and $-a^2$ have identical variables: a^2 . They are like terms. The other terms all have different variables.

The exponents on all variables must be identical for terms to be like.

- c) $3u^2$, $-4uv$, 6 , $2u^2$, $\frac{1}{2}v$, -5

$3u^2$ and $2u^2$ are like terms.

6 and -5 are also like terms.

$\frac{1}{2}v$ and $-4uv$ are unlike any of the other terms.

To simplify a polynomial expression, add or subtract like terms.

Example 3 Add Like Terms

Add.

a) $4x + 3x$

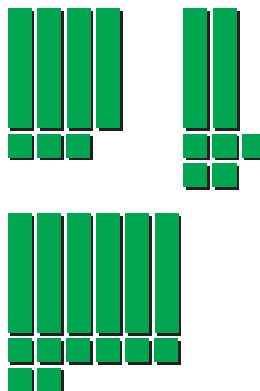
b) $4x + 3 + 2x + 5$

Solution

a) $4x + 3x$
 $= 7x$



b) $4x + 3 + 2x + 5$
 $= 4x + 2x + 3 + 5$
 $= 6x + 8$
 $= 6x + 8$



Example 4 Subtract Like Terms

Subtract.

a) $8x - 3x$

b) $2x - 5x$

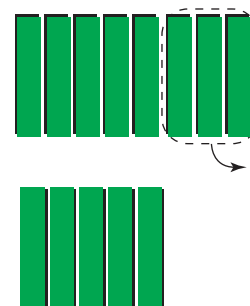
Solution

a) $8x - 3x$

Method 1: Take Away

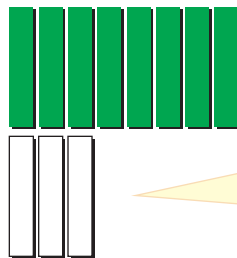
Start with 8 x-tiles. Remove 3 of them.

$8x - 3x$
 $= 5x$

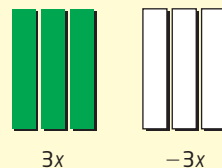


Method 2: Add the Opposite

Start with 8 x -tiles. Add 3 negative x -tiles.

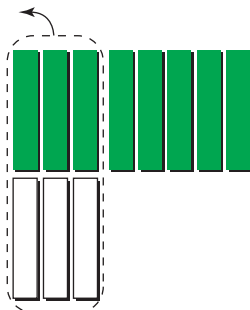


I use a different colour of tile to represent negative values.



Remove 3 zero pairs.

$3x$ and $-3x$ are opposites.
Their sum is 0, leaving $5x$ as the result.



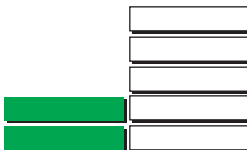
$$\begin{aligned} 8x - 3x \\ = 5x \end{aligned}$$



b) Start with $2x$ -tiles.

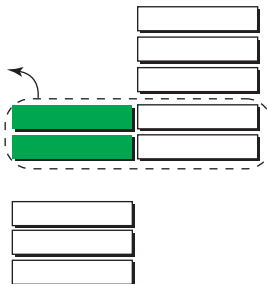


You cannot remove 5 x -tiles, because there are only 2 of them. You must add 5 $-x$ -tiles.



Remove 2 zero pairs.

$$\begin{aligned} 2x - 5x \\ = -3x \end{aligned}$$



Adding and subtracting to simplify algebraic expressions is called **collecting like terms**.

Example 5 Collect Like Terms With Concrete Materials

Use algebra tiles to simplify each expression.

a) $5k - 3k - 6 + 3$

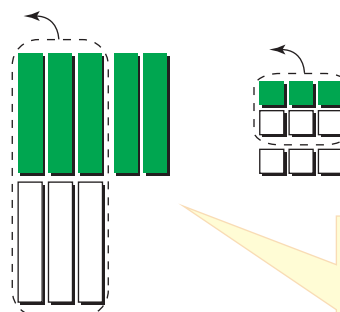
b) $3c^2 + c - 2 - 5c - 4c^2$

Solution

a) $5k - 3k - 6 + 3$



$= 5k - 3k - 6 + 3$



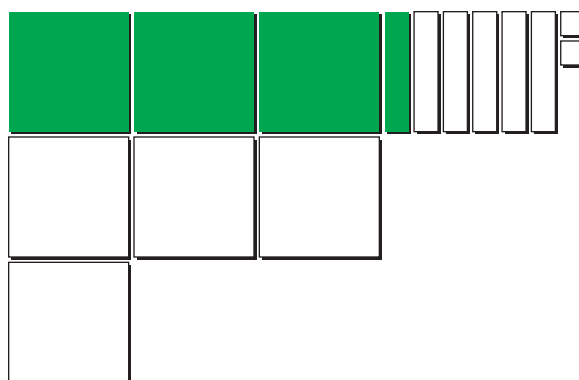
$= 2k - 3$



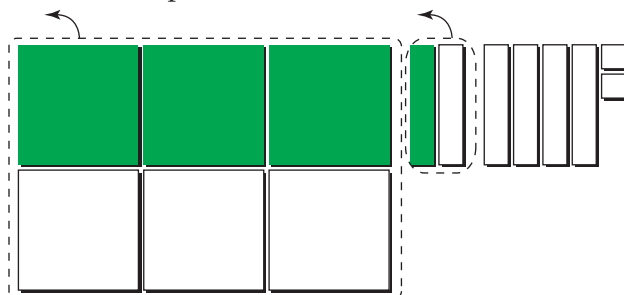
b) $3c^2 + c - 2 - 5c - 4c^2$



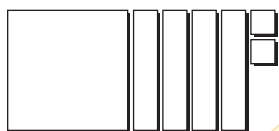
Rearrange the tiles to group like terms.



Remove zero pairs.



$$= 3c^2 - 4c^2 + c - 5c - 2$$



$$= -c^2 - 4c - 2$$

When the coefficient is 1 or -1 , you can omit the 1. But you must keep the minus sign.

$$\begin{aligned} 3c^2 - 4c^2 &= -1c^2 \\ &= -c^2 \end{aligned}$$

Algebraic expressions are usually written in order of degree, with the greatest-degree term first and the least-degree term last.

Example 6: Collect Like Terms Without Concrete Materials

Simplify.

- $6x + 4 + 8x + 3$
- $2b - b + 7 - 8$
- $3r^2 + 2 - 6r + 9r - 3r^2$
- $0.5m - 4.5n + 0.7m + 4.5n - 1.5$

Solution

When collecting like terms,

- group or identify like terms
- add or subtract like terms only
- apply integer rules to the coefficients of like terms
- do not change the variable parts

a) $6x + 4 + 8x + 3$

Identify like terms first.
 $6x$ and $8x$ are like terms.
 4 and 3 are also like terms.

Method 1: Align Like Terms Vertically

$$\begin{array}{r} 6x + 4 \\ + 8x + 3 \\ \hline 14x + 7 \end{array}$$

Method 2: Collect Like Terms

$$\begin{aligned} &6x + 4 + 8x + 3 \\ &= 6x + 8x + 4 + 3 \\ &= 14x + 7 \end{aligned}$$

Once you have had some practice, you may be able to skip this step.

Literacy Connections

When reading coefficients, always include the sign on the left. For example, the coefficient of $-b$ is -1 . Then, add the coefficients of like terms using integer rules. It is important that when you collect terms, you include the sign to the left of them:

$$2b - b + 7 - 8$$

b) $2b - b + 7 - 8$

Method 1: Align Like Terms Vertically

$$\begin{array}{r} 2b + 7 \\ + -b - 8 \\ \hline b - 1 \end{array}$$

$2b$ and $-b$ are like terms.
 7 and -8 are also like terms.

Apply integer rules to the coefficients of like terms:

$$2 + (-1) = 1$$

$$7 + (-8) = -1$$

Method 2: Collect Like Terms

$$\begin{aligned} 2b - b + 7 - 8 \\ = b - 1 \end{aligned}$$

c)
$$\begin{aligned} 3r^2 + 2 - 6r + 9r - 3r^2 \\ = 3r^2 - 3r^2 - 6r + 9r + 2 \\ = 3r + 2 \end{aligned}$$

d)
$$\begin{aligned} 0.5m - 4.5n + 0.7m + 4.5n - 1.5 \\ = 0.5m + 0.7m - 4.5n + 4.5n - 1.5 \\ = 1.2m - 1.5 \end{aligned}$$

$-4.5n$ and $4.5n$ are opposites. They add to 0.

Key Concepts

- Like terms have identical variables.
- You can simplify a polynomial by adding or subtracting like terms. This is called collecting like terms.
- It is helpful to think of only adding terms. When a subtraction sign appears, think of adding a negative term.

Communicate Your Understanding

- C1** a) Explain what is meant by *like terms* and *unlike terms*.
b) Provide two examples of each.
- C2** Explain why the two sides are not equal.
- a) $2x + x + 3x \neq 5x$
 - b) $y + y + y \neq y^3$
 - c) $-5m + 2m \neq 3m$
 - d) $-2x - 2x \neq 0$
 - e) $x + x \neq x^2$
 - f) $3ab - 2b \neq a$

Practise

For help with questions 1 to 4, see Example 2.

- Which polynomial contains no like terms?
A $2x + 5 - 3x + 2xy$ **B** $3x^2 + 3xy + 3$
C $4 - 9x + 9y + 3$ **D** $-4a^3 + 5b - 2a^2 + 7b$
- Classify each pair of terms as either like or unlike.
a) $2x$ and $-5x$ **b)** $3y$ and $3z$ **c)** $-x^2$ and $\frac{1}{2}x^2$
d) $4a^2$ and $3a^3$ **e)** $2ab$ and $3a^2$ **f)** $5x^2y$ and $-2xy^2$
g) $3uv$ and $2vu$ **h)** $9p^2q^3$ and $-4q^3p^2$
- Copy the two columns of terms into your notebook. Connect each term in the first column with the like term in the second column.

$3x^2$	$-4xy$
$-x$	y^2
xy	$\frac{1}{2}y$
$2y^2$	$7x$
$5x^2y$	$4x^2$
$-7y$	$3x^2y$

- Write two like terms for each.
a) $5m$ **b)** $-x$ **c)** $2y^2$ **d)** $3ab$

For help with question 5, see Example 3.

- Simplify where possible. If it is not possible to simplify, explain why.
a) $3x + 6x$ **b)** $2m + 5n$
c) $5h + 8h + 2h$ **d)** $7u + 4u + u$

For help with question 6, see Example 4.

- Simplify. If it is not possible to simplify, explain why.
a) $4k - 2k$ **b)** $8n - n$
c) $3z - 7z$ **d)** $p - 6$

For help with questions 7 to 9, see Example 5.

- Simplify by collecting like terms.
a) $3x + 5 + 2x + 1$ **b)** $3 + 7y + 8 + y$
c) $2k + 3m + 4m + 6k$ **d)** $7u + v + v + u$
e) $8n + 5 - 3n - 2$ **f)** $4p + 7q - 3q - p$

Making Connections

Algebraic expressions are used in work with spreadsheets. You use formulas in cells to tell the computer to make specified calculations. For example, the formula in cell D2 might be $2 \times B2 + 2 \times C2$. You will have the opportunity to apply this connection in Chapter 9 Optimizing Measurement.

8. Simplify.

- a) $3x - 8 - 4x + 3$
- b) $y - 9 - 7 - 6y$
- c) $2x^2 + 7x + 4x^2 + x$
- d) $7m + 6m^2 - 2m + m^2$
- e) $3k - 5 + 8 - k + 1 - 4k$
- f) $-3u + 2 - u^2 - 5 + 3u + 2u^2 - 3$

9. Simplify.

- a) $2a^2 - 3ab - 6 + 4b^2 + 7 + 5ab - 3b - 2a^2$
- b) $3mn + 6m^2 - n^2 + 3 - m^2 - 3mn + 2n^2 - 4$

Connect and Apply

10. Claudette, Johanna, and Ming all have summer jobs at a clothing store. They earn \$7/h, plus a bonus every week that depends on how well they do that week. The table shows how many hours they work and their bonuses one week this summer.

Employee	Number of Hours	Bonus
Claudette	20	\$100
Johanna	25	\$125
Ming	27	\$110

- a) Using t to represent the time, in hours, write a binomial to represent the amount each student makes that week.
 - b) Substitute the time, in hours, into each expression, and find how much each student makes.
 - c) The three students decide to put their earnings for that week together to go for an end-of-summer trip. Write a binomial to represent the total amount they earn that week.
 - d) Substitute the total number of hours and find the total amount the students make that week. Compare your answer with what you found in part b).
11. Yannick simplified the following expression:
- $$3x + 4 + 6x = 13x$$
- a) Describe the error Yannick made.
 - b) How can you convince Yannick that these two expressions are not equal?
 - c) Simplify the expression properly. How can you convince Yannick that your answer is correct?

12. The length of a rectangular field is three times its width.
- Write an expression for the perimeter of the field.
 - Find the perimeter if the field is 300 m wide.
 - Find the length and width of the field if the perimeter is 1600 m.



13. Use algebra tiles, virtual algebra tiles, or a diagram to model and simplify each expression.

- $3x + 1 + 5x + 4$
- $4y + 3 - y - 2$
- $x + 5y + 8 - x - 2y + 2$

14. **Chapter Problem** The white bars from the right of the crest into the centre run halfway across the width.
- Find an expression for the
 - perimeter
 - total length of white trim needed
 - What length of trim will be needed for a crest that is 10 cm wide?



Extend

15. **a)** An equilateral triangle has an unknown side length, x . Write a simplified expression for its perimeter.
- b)** A right isosceles triangle has two sides equal to x . Which triangle, the equilateral triangle in part a) or the right isosceles triangle, has the greater perimeter? Use algebraic reasoning to justify your answer.
16. Which triangle in question 15 has the greater area? Use algebraic reasoning to justify your answer.

17. **Math Contest** What is the value of the 100th term in the sequence $3x + 2y, 5x^2 + 5y^3, 7x^3 + 8y^5, \dots$?

- $199x^{100} + 296y^{197}$
- $200x^{100} + 300y^{199}$
- $200x^{100} + 300y^{197}$
- $201x^{100} + 299y^{199}$
- $203x^{100} + 299y^{199}$

18. **Math Contest** The last digit of the number 2^{2020} when written in expanded form is

- | | | |
|------------|------------|------------|
| A 2 | B 4 | C 6 |
| D 8 | E 0 | |